Service System Design in the Public and Private Sectors

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1. Introduction

Needs and requirements of human society or particular social groups form various demands, which are usually spread over a geographical area. An effective satisfaction of the demands is possible only if the corresponding service provider concentrates its sources at several places of the served area and if he provides the service in these places only or if he serves the demands at their positions by trips starting and terminating at these places. To denote source of a service, we shall use the term "facility", which is mend to include a broad set of entities such as factories, warehouses, transportation terminals, schools, hospitals, day-care centres, public administration offices, emergency warning sirens and others. An addressee of the service will be denoted by the term "customer", even if he has hardly anything common with this term in the market sense in many cases.

Within frame of this paper we restrict ourselves on such problems, in which a finite number of customers and finite number of possible facility locations are considered, what could be pretty good approximation of most of real cases. The great deal of a service system design will be done, when question on a number of facilities and their locations are answered. We shall distinguish two classes of service systems, which differ in objectives. The first considered class is formed by so-called public systems and the second one is referred as private systems. When a system from the first class is designed, the objective is stated as minimization of a social cost subject to service each customer or, in addition, some customer's equity in access to the service may be demanded. On the contrary to a public systems design, a private system designer accents profit maximization or capture of larger market share. In this case, service of some disadvantageous demands can be omitted.

In the following sections we formulate several typical models of both private and public system designs and we shall discuss means of corresponding solving approaches. Then we show a way, how to rearrange all reported models to general one, problem instances of which are solvable by exact algorithms for considerable large size.

2. Public Service Systems

As preliminaries for model construction, we introduce the following notation of particular terms, which will be used throughout the whole paper. Let J denote a finite set of customers and if a quantity of customer's demand can be expressed by a real number, then demand of customer $j \in J$ be denoted by b_j. Let I denote a finite set of possible facility locations and let d_{ij} denote the distance between location $i \in I$ and the location of customer $j \in J$. The decision on facility location at place $i \in I$ is modelled by zero-one variable $y_i \in \{0,1\}$, which takes value 1 if a facility should be located at i and it takes value 0 otherwise.

2.1 Maximum Distance Model

In this problem a customer's demand is covered if its distance from some located facility is less or equal than given constant D. This case corresponds to problems like emergency warning sirens locations or health centre location. The objective is to cover all the customer demands with minimum number of located facilities. The classical approach [Current, 2002], [Marianov, 2002] introduces set $N_j=\{i \in I: d_{ij} \le D\}$ of possible locations, from which demand of customer j can be satisfied. Then the model of the corresponding set covering problem can be established in the form:

$$\text{Minimize} \quad \sum_{i \in I} y_i \qquad (1)$$

$$\text{Subject to} \quad \sum_{i \in N_j} y_i \ge 1 \qquad \text{for } j \in J. \qquad (2)$$

2.2 p-Centre Problem

This problem consists in minimizing the maximum distance between customer and the nearest located facility, when pre-determined number of p facilities is given. This problem arises, when limited number of fire-stations or first-aid stations should be located so that time, in which the worst situated customer can be served, be as small as possible. To form a model of this problem, auxiliary variables $z_{ij} \in \{0,1\}$ for each $i \in I$ and $j \in J$ are introduced to assign customer j to possible location i. Furthermore, we add an another variable $w \ge 0$, which is used as upper bound of the distances between customer j and assigned location i. The p-center problem can be formulated as follows:

$$\text{Minimize} \quad w \qquad (3)$$

$$\text{Subject to} \quad \sum_{i \in I} z_{ij} = 1 \qquad \text{for } j \in J \qquad (4)$$

$$z_{ij} \le y_i \qquad \text{for } i \in I \text{ and } j \in J \qquad (5)$$

$$\sum_{i \in I} y_i = p \qquad (6)$$

$$\sum_{i\in I} d_{ij}\, z_{ij} \le w \qquad \text{for } j\in J. \qquad (7)$$

2.3 p-Median Problem

A problem of this type arises, when the number of facilities is fixed and average distance or average weighted distance between customer and the nearest located facility should be minimized. In the public sector, one might want to locate, for example, public administration bureaus in such a way as to minimize the total distance that citizens must traverse to reach their closest bureau. If J denotes set of dwelling places in the area, b_j number of inhabitants at $j\in J$ and if we use the previously introduced variables, an associated model to this problem can be established this way:

$$\text{Minimize} \quad \sum_{i\in I}\sum_{j\in J} b_j d_{ij}\, z_{ij} \qquad (8)$$

Subject to (4), (5), (6).

3. Private Service Systems

Making use of above introduced notation, we try to formulate models of two broadly spread problems in the private sector.

3.1 Maximum Covering Location Problem

Similarly to the previous models, the number of facilities is fixed at value p, but, on the contrary to public the systems, not each customer must be served. We consider that service of customer j brings profit, which is proportional to its demand b_j. The customer is considered to be served if there is located at least one facility within distance D from the customer. Making use of sets N_j introduced in sub-section 2.1 and introducing auxiliary variables $x_j\in\{0,1\}$ taking value 1, if customer j is served, we can established the following model:

$$\text{Maximize} \quad \sum_{j\in J} b_j\, x_j \qquad (9)$$

$$\text{Subject to} \quad \sum_{i\in N_j} y_i \ge x_j \qquad \text{for } j\in J \qquad (10)$$

$$\sum_{i\in I} y_i = p. \qquad (11)$$

3.2 Fixed Charge Location Problem

We consider a simpler version of the problem stated in [Current, 2002] or [Kubanová, 1997]. The number of facilities in not fixed here, but each facility location at $i\in I$ is connected with fixed charge f_i, which doesn’t depend on demand quantity satisfied from this location. Furthermore, costs c_{ij} are introduced to express cost connected with servicing customer j from a facility located at i. The objective is to satisfy all customer demands and to minimize the total costs including both fixed charges and service costs. This problem often emerges, when a distribution system is designed. Making use of previously introduced variables y_i and z_{ij}, the model can be stated as follows:

$$\text{Minimize} \quad \sum_{i \in I} f_i \, y_i + \sum_{i \in I} \sum_{j \in J} c_{ij} \, z_{ij} \qquad (12)$$

Subject to *(4), (5).*

4. Solving Methods

There are various exact methods for solving the particular problems mentioned above [Buzna, 2003], [Current, 2002], [Erlenkotter, 1978] [Marianov, 2002],. Besides, having formulated linear mathematical programming model, some of general solvers based on branch and bound approach can be used [Jablonský, 2002]. As shown in [Chocholácek, 1998] for particular problem, special approaches win from the time consumption point of view, if special and general solver approaches are compared. Neverless, if the studied problem is modified e.g. by addition of a new constraint or by slight change in the objective function, then these special approaches are useless.

That is why we focus on the possibility, how to rearrange the broad spectrum of problems to general one, for which a smart solving tool has been developed. The general problem has form of the above fixed charge location problem with limited number of used facilities. The model has the following form:

$$\text{Minimize} \quad \sum_{i \in I} f_i \, y_i + \sum_{i \in I} \sum_{j \in J} c_{ij} \, z_{ij} \qquad (13)$$

$$\text{Subject to} \quad \sum_{i \in I} z_{ij} = 1 \qquad \text{for } j \in J \qquad (14)$$

$$z_{ij} \le y_i \qquad \text{for } i \in I \text{ and } j \in J \qquad (15)$$

$$\sum_{i \in I} y_i \le p \qquad (16)$$

The associated problem can be solved by the approach reported in [Janácek, 2000], where Lagrangean multiplier f is introduced for constraint *(16)*, which is to be relaxed. Then the problem *(14)-(16)* can be reformulated this way: Find $f \ge 0$, so that values of variables y_i , $i \in I$ of the optimal solution of problem *(17)*, *(14)*, *(15)* meet constraint *(16)* as equality. The considered objective function is

$$\sum_{i \in I} (f + f_i) y_i + \sum_{i \in I} \sum_{j \in J} c_{ij} x_{ij} \, . \qquad (17)$$

If f is fixed, then problem *(17), (14), (15)* forms an uncapacitated location problem. To solve the problem for nonnegative values of f and $\{c_{ij}\}$, procedure *BBDual* [Janácek, 1997] was devised and implemented. Being tested during computational experiments with large networks, the procedure proved to be able to solve large size problems quickly enough to be used repeatedly in more complicated algorithms.

To find demanded value f, an algorithm was completed [Janácek, 2000], in which function $Q(f, \mathbf{c})$ gives number of variables y_i which value is equal to one in the optimal solution of problem *(17), (14), (15)* for given $f, \mathbf{c}$.

0. **Set** $f_{min} = 0, f_{max} = \sum_{j \in J} max\{ c_{ij} : i \in I \}, f = (f_{max} - f_{min})/2$.

1. **While** $(Q(f, \mathbf{c}) \neq p)$ *and* $(f_{max} - f_{min} \geq \varepsilon)$ **repeat**

   If $Q(f, \mathbf{c}) > p$ **then set** $f_{min} = f$, **otherwise set** $f_{max} = f$.

   Set $f = (f_{max} - f_{min})/2$.

It is necessary to remark that the optimal solution $< \mathbf{y}, \mathbf{z} >$ of problem *(17), (14), (15)* for resulting f need not necessarily meet constraint *(16)* as equality [Janácek, 2000].

5. Problem Rearrangements

In this section, there will be shown that it is possible to rearrange each of the above models to the form of model *(13)-(16)*. The reformulation is not necessary for the fixed charge location problem (3.2). In that case, it is sufficient to omit the constraint, which bounds the number of used facilities. Furthermore, the p-median problem (2.3) with nonnegative coefficients can be considered as a form of the generalized problem for $f_i=0$.

The rearrangement of models 2.1, 2.2 and 3.1 can be done by the following way:

Maximum Distance Model (2.1) can be reformulated to model *(13)-(16)* defining $f_i=1$ for $i \in I$ and $c_{ij}=0$ for $j \in J$ and $i \in N_j$ and $c_{ij}=2$ otherwise.

p-Centre Problem (2.2) cannot be transformed directly to model *(13)-(16)*, but such a model can be derived that its solution provides approximate solution of the p-centre problem and by repeating the solving process, the resulting solution can be made arbitrary precise. To reach this goal, lower and upper bounds on optimal value of w must be determined. Let us denote the current bounds w_{min} and w_{max}. The interval $[w_{min}, w_{max}]$ is divided into r equidistant parts by values $w_1 < w_2 < \ldots < w_r = w_{max}$. Then the surrogate costs c_{ij} for $j \in J$ and $i \in I$ are established in accordance to the rule: $c_{ij}=0$ for $d_{ij}<w_1$; $c_{ij}=(|J|-p)^k$ for $w_k \leq d_{ij}<w_{k+1}$ for $k=1,..r-1$; $c_{ij}=(|J|-p)^r$ for $w_r \leq d_{ij}$. With this costs p-median problem can be solved and its

largest value c_{ij}, for which optimal $z_{ij}=1$ determines $\underline{k}$ and associated $w_{\underline{k}}$, $w_{\underline{k}+1}$ form new lower and upper bounds on the original problem.

Maximum Covering Location Problem (3.1)

To rearrange the former model, we introduce assignment variables $z_{ij} \in \{0, 1\}$ taking value 1 if and only if customer j is assigned to place $i \in I$. Then we can rewrite the former model as:

$$\text{Maximize} \quad \sum_{j \in J} \sum_{i \in N_j} b_j z_{ij} \qquad (18)$$

$$\text{Subject to} \quad \sum_{i \in N_j} z_{ij} \le 1 \qquad \text{for } j \in J \qquad (19)$$

$$z_{ij} \le y_i \qquad \text{for } j \in J \text{ and } i \in N_j \qquad (20)$$

$$\sum_{i \in I} y_i = p \qquad (21)$$

Further we add one "fictive" place i_0 to each neighbourhood N_j obtaining new neighbourhoods $\underline{N}_j=N_j \cup \{i_0\}$. Now we introduce slack variables z_{i0j} for each constraint *(19)* and defining $\underline{c}_{ij}=b_j$ for each $j \in J$ and $i \in N_j$ and $\underline{c}_{i0j}=0$ for each $j \in J$, we obtain model, in which constrains *(19)* take form of equality.

After this arrangement, constant $|J|*\underline{C} = |J|*max\{\underline{c}_{ij}: j \in J, i \in \underline{N}_j\}=\underline{C}*\sum_{j \in J} \sum_{i \in \underline{N}_j} z_{ij}$ can be subtracted from the *(18)* without loss of generality. This way, a new objective function with non-positive coefficients $(\underline{c}_{ij}-\underline{C})$ is obtained and when maximization is replaced with minimization of the objective function with nonnegative coefficients $c_{ij}=\underline{C}-\underline{c}_{ij}$, than the only difference between the general model and the obtained one consists in summation over sets $\underline{N}_j$. This can be adjusted by introducing some prohibitive constant $C>\underline{C}$ and coefficients $c_{ij}=C$ for $j \in J$ and $i \notin \underline{N}_j$ together with the associated variables z_{ij}.

6. Conclusions

We have shown that a broad spectrum of location problems originating in both private and public sectors can be rearranged to the form of uncapacitated location problem with simple constraint on number of located facilities. Furthermore, an approximative approach based on uncapacitated location technique was referred, which very often reaches an optimal solution. An advantage of the suggested solving process consists in speed, with which it is possible to obtain an exact solution of a very large problem in comparison with general integer programming solvers. Results reported in this contribution accents importance of further development of exact solving techniques for uncapacitated location problem on various types of underlying networks. Unfortunately, our preliminary experience indicates that if capacitated constraints are considered in a location problem, its intractability increases and disables exact solving of large instances. Overcoming of this obstacle may represent a topic of a further research.

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