

SUPPLEMENTARY INFORMATION

A Versatile Adaptive Aggregation Framework for Spatially Large Discrete Location-Allocation Problems

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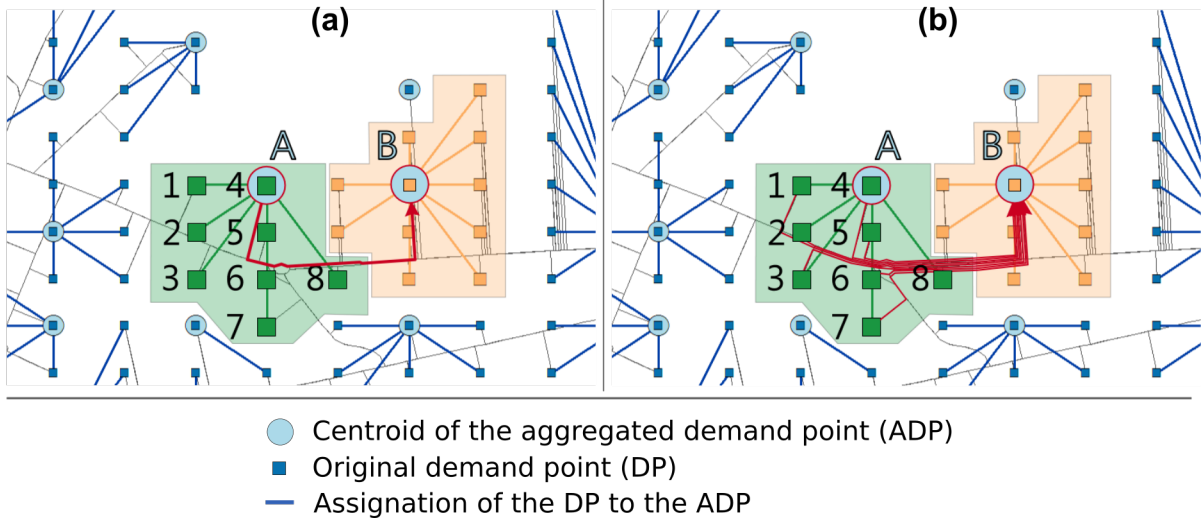
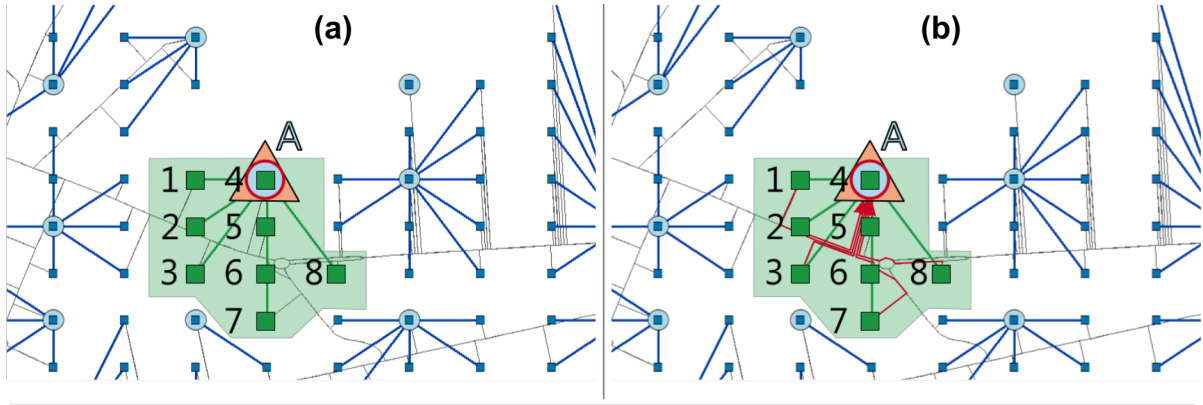
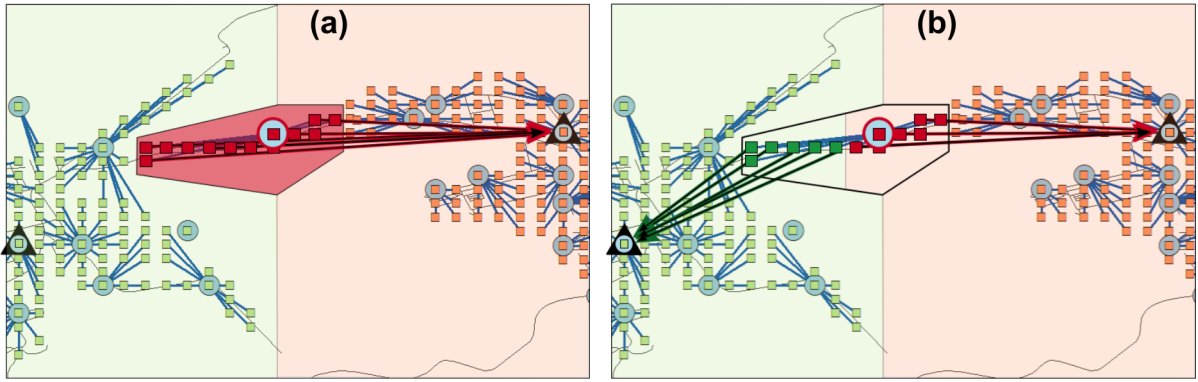


Figure S1 Illustration of the source error A. Green and orange polygons represent ADP A (customer locations) and ADP B (possible facility location), respectively. Each ADP is composed of DPs, shown as squares, and is represented by the centroid visualized by the large circle. (a) Usually the distance between ADP A and ADP B is measured as the shortest path length in the underlying graph of the road network connecting two centroids (see the red polyline leading from centroid A to the centroid B). Thus, the original DPs are not considered. (b) Elimination of the source error A is based on considering the distance from A to B to be the weighted sum of all shortest path lengths from all DPs represented by the centroid A to the centroid B, i.e., $\sum_{i=1}^8 d_{iB} w_i$, where d_{iB} is the shortest path length from the DP i to the centroid B and w_i is the weight assigned to the DP i (see the red polylines leading from DPs that constitute the centroid A to the centroid B).



- ▲ Located Facility
- Centroid of the aggregated demand point (ADP)
- Original demand point (DP)
- Assignment of the DP to the ADP

Figure S2 (a) Illustration of the source error B. Often, if a facility is located in ADP, then zero distance is considered between customers that are represented by the ADP and the possible facility location that is represented by the centroid of ADP. (b) Elimination of the source error B is based on considering sum of all weighted distances from all DPs represented by the centroid A to the centroid A $\sum_{i=1}^8 d_{iA} w_i$, where d_{iA} is distance from DP i to centroid A and w_i is the weight assigned to the DP i .



- ▲ Located Facility
- ▭ Service area of the facility
- Centroid of the aggregated demand point (ADP)
- Original demand point (DP)
- Assignment of the DP to the ADP

Figure S3 (a) Illustration of the source error C. Usually, all DPs that are represented by an ADP are assigned to the same facility as their centroid (illustrated by the arrows pointing to the facility located on the right). (b) Elimination of the source error C is based on the re-allocation of DPs to the closest facility. Used arrows indicate that one group of DPs is assigned to the facility located on the left and the second group is assigned to the located facility situated on the right.

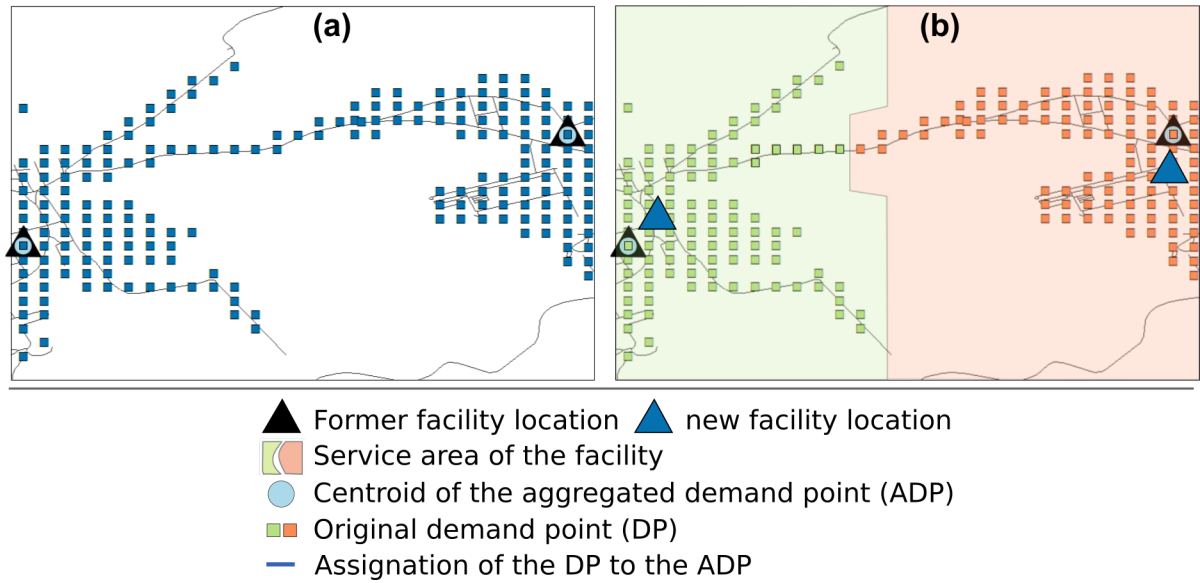


Figure S4 Illustration of the source error D. (a) When problem data are aggregated, facilities are located in the centroids of some ADP and not in DPs. Thus, locations of facilities is not optimal regarding the original problem that consists from all DPs. (b) One possible way how to minimize source error D is to define a service zone for each located facility as a set of DPs that are allocated to it (in the figure indicated by the color of squares that represent DPs). For each service zone we solve location problem while looking for the optimal position of one facility. The optimal facility location is considered as the new facility location that is minimizing the source error D.

$\hat{\epsilon}$ [unit]	$\hat{\alpha}_1$ [%]	Indicator	$\hat{p}=5$			$\hat{p}=10$			$\hat{p}=20$		
			V1	V2	V3	V1	V2	V3	V1	V2	V3
0	99	$\alpha_{i_{last}} [\%] i_{last}$	95.29	88.3 8	85.6 9	91.1 9	83.5 8	81.0 10	81.5 12	67.3 12	71.3 8
		$\Phi(0, \alpha_{i_{last}} [\%])$	0.829	0.266	0.05	0.81	0.488	0	0.738	0.289	0.091
		$\tau(0, \alpha_{i_{last}} [\%])$	-94.7	-81.3	-62.3	-64.9	-50.1	-2.4	-56.1	-21.7	-10.1
	90	$\alpha_{i_{last}} [\%] i_{last}$	89.3 5	86.1 5	85.1 5	87.6 6	81.2 6	83.5 7	81.2 5	72.2 7	72.4 8
		$\Phi(0, \alpha_{i_{last}} [\%])$	0.024	0.058	0.05	0.894	0.401	0.044	0.891	0.54	0.029
		$\tau(0, \alpha_{i_{last}} [\%])$	-91.8	-81.8	-71.1	-72	-52.9	-36.1	-72.4	-42.3	-14.1
	75	$\alpha_{i_{last}} [\%] i_{last}$	78.0 4	75.7 6	75.7 5	77.2 4	74.1 5	72.6 8	63.0 3	65.0 8	68.1 6
		$\Phi(0, \alpha_{i_{last}} [\%])$	0.044	0.024	0.02	0.7	0.708	0.003	0.14	0.429	0.045
		$\tau(0, \alpha_{i_{last}} [\%])$	-87.5	-69.6	-42.8	-57.9	-39.2	23.9	-64.2	-32.4	2.1
100	99	$\alpha_{i_{last}} [\%] i_{last}$	93.8 9	86.6 8	83.8 9	87.5 11	80.8 8	75.8 10	66.3 18	58.8 11	62.1 9
		$\Phi(0, \alpha_{i_{last}} [\%])$	0.829	0.266	0.04	0.427	0.369	0	0.445	0.4	0.07
		$\tau(0, \alpha_{i_{last}} [\%])$	-88.2	-76.6	-53.3	-52.8	-44	28	12.6	13.8	59.4
	90	$\alpha_{i_{last}} [\%] i_{last}$	86.9 6	82.6 6	82.5 6	81.0 8	73.6 9	77.9 7	67.8 6	58.5 11	61.0 9
		$\Phi(0, \alpha_{i_{last}} [\%])$	0.062	0.04	0.042	0.793	0.29	0.003	0.85	0.334	0.016
		$\tau(0, \alpha_{i_{last}} [\%])$	-79	-71	-60.2	-32.8	-4.6	-4.3	-39.1	30.1	59.8
	75	$\alpha_{i_{last}} [\%] i_{last}$	75.7 5	73.5 5	73.3 5	71.1 6	66.4 7	70.4 5	64.1 7	55.4 8	57.0 6
		$\Phi(0, \alpha_{i_{last}} [\%])$	0.03	0.03	0.014	0.694	0.656	0.078	0.384	0.214	0.016
		$\tau(0, \alpha_{i_{last}} [\%])$	-72.6	-64.9	-23.8	39.9	20.3	6.9	-27.4	12.2	44.6
200	99	$\alpha_{i_{last}} [\%] i_{last}$	88.5 9	80.8 9	71.3 10	61.1 11	51.4 10	48.0 16	22.4 11	20.0 9	24.3 17
		$\Phi(0, \alpha_{i_{last}} [\%])$	0.348	0.159	0.04	0.38	0.131	0	0.136	0.066	0.037
		$\tau(0, \alpha_{i_{last}} [\%])$	-81.5	-71.9	36.6	32.6	136.5	533.7	241.6	256.7	633.4
	90	$\alpha_{i_{last}} [\%] i_{last}$	70.0 8	68.1 7	69.0 8	49.5 6	53.9 7	53.0 13	22.5 7	17.1 8	22.5 15
		$\Phi(0, \alpha_{i_{last}} [\%])$	0.189	0.024	0.042	0.078	0.273	0.003	0.235	0.033	0.016
		$\tau(0, \alpha_{i_{last}} [\%])$	-43.1	-43.4	48.8	138.4	107.4	346.8	271.1	218.1	538
	75	$\alpha_{i_{last}} [\%] i_{last}$	64.5 5	62.5 5	61.3 18	42.4 8	42.5 7	49.5 14	18.6 7	14.0 7	20.7 18
		$\Phi(0, \alpha_{i_{last}} [\%])$	0.066	0.03	0.014	0.537	0.067	0.078	0.148	0.033	0.016
		$\tau(0, \alpha_{i_{last}} [\%])$	-47.6	-43.3	441.3	290.4	281.3	1060.2	519.8	264.9	783.3

Table S1 Results of numerical experiments after applying the re-aggregation framework to the p-median problem and the benchmark d2103. By the symbol $\alpha_{i_{last}}$ we denote the aggregation level after accomplishing the last iteration i_{last} of the framework.

$\hat{\epsilon}$ [unit]	$\hat{\alpha}_1$ [%]	Indicator	$\hat{p}=5$			$\hat{p}=10$			$\hat{p}=20$		
			V1	V2	V3	V1	V2	V3	V1	V2	V3
0	99	$\alpha_{i_{last}} [\%] i_{last}$	96.8 7	89.1 8	85.6 9	93.2 10	79.1 9	79.9 7	86.8 10	68.2 8	69.6 8
		$\Phi(0, \alpha_{i_{last}} [\%])$	0.888	0.274	0.049	0.837	0.129	0	1.192	0.459	0.109
		$\tau(0, \alpha_{i_{last}} [\%])$	-97.5	-95.1	-90.1	-93.3	-81	-79.9	-81.9	-63.5	-54.8
	90	$\alpha_{i_{last}} [\%] i_{last}$	89.4 4	82.5 7	83.0 6	88.8 3	77.4 7	76.8 5	84.4 9	67.6 8	70.8 6
		$\Phi(0, \alpha_{i_{last}} [\%])$	0.129	0.064	0	0.267	0.254	0.063	0.702	0.421	0.117
		$\tau(0, \alpha_{i_{last}} [\%])$	-91.5	-89.3	-86	-85.6	-68.4	-67.2	-58.7	-45	-45.2
	75	$\alpha_{i_{last}} [\%] i_{last}$	75.1 3	73.3 4	73.3 4	74.5 3	70.1 4	70.2 4	72.9 4	66.3 5	64.0 6
		$\Phi(0, \alpha_{i_{last}} [\%])$	0.017	0.017	0.017	0.005	0.02	0.018	0.378	0.411	0.176
		$\tau(0, \alpha_{i_{last}} [\%])$	-86	-83.8	-62	-59.8	-54.3	-47.9	-8.9	-33.6	13.7
100	99	$\alpha_{i_{last}} [\%] i_{last}$	96.2 8	87.7 8	82.2 9	91.7 11	76.7 10	75.5 9	74.9 16	56.9 11	60.6 11
		$\Phi(0, \alpha_{i_{last}} [\%])$	0.888	0.316	0	0.671	0.129	0.01	0.595	0.266	0.031
		$\tau(0, \alpha_{i_{last}} [\%])$	-96.8	-94.6	-86.5	-87.7	-77.3	-67.5	-41.4	-25.4	-2.6
	90	$\alpha_{i_{last}} [\%] i_{last}$	88.9 4	82.3 6	81.2 6	86.3 4	76.9 6	73.5 6	75.9 12	57.7 10	58.1 9
		$\Phi(0, \alpha_{i_{last}} [\%])$	0.064	0.064	0	0.267	0.272	0.063	0.704	0.323	0.021
		$\tau(0, \alpha_{i_{last}} [\%])$	-91.6	-89.3	-83.7	-74	-73.3	-59	-31.3	-11.5	25
	75	$\alpha_{i_{last}} [\%] i_{last}$	74.3 4	72.5 4	72.0 5	70.9 5	65.9 6	66.6 6	61.9 7	52.3 6	54.2 8
		$\Phi(0, \alpha_{i_{last}} [\%])$	0.017	0.017	0.002	0.112	0.013	0.007	0.202	0.233	0.017
		$\tau(0, \alpha_{i_{last}} [\%])$	-83.5	-83.5	-59.6	-47.9	-49.2	-13.4	25.9	48.1	64.2
200	99	$\alpha_{i_{last}} [\%] i_{last}$	91.2 10	83.3 7	75.6 10	79.9 13	64.7 11	62.1 8	33.6 19	22.7 12	35.0 10
		$\Phi(0, \alpha_{i_{last}} [\%])$	0.397	0.171	0	1.144	0.053	0.01	0.58	0.093	0.007
		$\tau(0, \alpha_{i_{last}} [\%])$	-95.7	-94.6	-73.2	-78.3	-55.5	-36.7	495.1	894	145.7
	90	$\alpha_{i_{last}} [\%] i_{last}$	82.2 7	74.4 8	74.4 7	69.7 7	60.1 7	59.2 8	42.5 8	30.6 8	34.5 6
		$\Phi(0, \alpha_{i_{last}} [\%])$	0.038	0.038	0	0.089	0.17	0.007	0.45	0.055	0.019
		$\tau(0, \alpha_{i_{last}} [\%])$	-88.7	-84.4	-68.3	-52	-44.3	4.7	64.9	72.5	70.7
	75	$\alpha_{i_{last}} [\%] i_{last}$	70.1 4	68.4 4	67.2 6	60.4 4	55.3 5	55.5 6	35.0 7	29.0 6	33.3 5
		$\Phi(0, \alpha_{i_{last}} [\%])$	0.017	0.017	0.002	0.049	0.013	0.007	0.088	0.069	0.024
		$\tau(0, \alpha_{i_{last}} [\%])$	-81.3	-82.4	-36.8	-33.2	-25.7	30.6	158.1	80.9	101.2

Table S2 Results of numerical experiments after applying the re-aggregation framework to the p-median problem and the benchmark pcb3038.

$\hat{\epsilon}$ [km]	$\hat{\alpha}_1$ [%]	Indicator	$\hat{p}=5$			$\hat{p}=10$			$\hat{p}=20$		
			V1	V2	V3	V1	V2	V3	V1	V2	V3
0	99	$\alpha_{i_{last}} [\%] i_{last}$	97.9 7	95.3 9	94.8 9	96.3 9	91.0 12	89.8 11	93.1 11	86.2 14	84.1 10
		$\Phi(0, \alpha_{i_{last}} [\%])$	1.237	0.772	0.173	3.519	1.831	0	3.654	1.566	0.105
		$\tau(0, \alpha_{i_{last}} [\%])$	-99.9	-99.6	-97.5	-99.6	-96.4	-93.5	-97.2	-71.7	-63.7
	90	$\alpha_{i_{last}} [\%] i_{last}$	90.5 4	89.3 6	89.1 5	86.8 8	86.8 7	87.1 6	88.6 5	83.0 12	83.3 8
		$\Phi(0, \alpha_{i_{last}} [\%])$	0.477	0.2	0.173	0.934	0.913	0.365	2.619	1.187	0.017
		$\tau(0, \alpha_{i_{last}} [\%])$	-99.4	-98.6	-96.3	-96.2	-94.7	-92.9	-96.5	-64.4	-61.1
	75	$\alpha_{i_{last}} [\%] i_{last}$	77.6 3	77.0 5	77.1 5	77.2 5	75.6 8	75.7 5	76.5 4	74.7 6	73.7 6
		$\Phi(0, \alpha_{i_{last}} [\%])$	0.037	0.037	0	0.644	0.529	0.043	0.883	1.14	0.017
		$\tau(0, \alpha_{i_{last}} [\%])$	-97.7	-95.8	-81.4	-95.7	-90.1	-85.8	-87.6	-72.3	-54.4
1	99	$\alpha_{i_{last}} [\%] i_{last}$	85.9 10	77.9 14	79.3 12	62.9 13	62.9 13	62.9 12	53.4 13	42.3 12	46.9 12
		$\Phi(0, \alpha_{i_{last}} [\%])$	0.567	0.235	0.173	0.087	0.087	0	0.393	0.115	0.105
		$\tau(0, \alpha_{i_{last}} [\%])$	-99.2	-93	-80.2	-81	-67.1	-55.1	-21.4	125.1	153.8
	90	$\alpha_{i_{last}} [\%] i_{last}$	75.4 7	74.0 11	75.2 11	62.8 11	62.8 11	62.5 11	48.8 8	43.6 11	44.8 11
		$\Phi(0, \alpha_{i_{last}} [\%])$	0.173	0.173	0.173	0.151	0.151	0	0.244	0.116	0.105
		$\tau(0, \alpha_{i_{last}} [\%])$	-96.7	-87.9	-61.4	-75.3	-64.8	-25.9	-35.4	156.4	223.6
	75	$\alpha_{i_{last}} [\%] i_{last}$	67.8 5	65.9 10	66.2 10	58.5 6	55.6 10	56.6 10	44.8 6	41.1 10	41.4 10
		$\Phi(0, \alpha_{i_{last}} [\%])$	0	0	0	0.003	0.003	0	0.049	0.049	0
		$\tau(0, \alpha_{i_{last}} [\%])$	-93.8	-82.5	-37.3	-88.2	-65.3	-33.7	-34.6	66	133.4
2	99	$\alpha_{i_{last}} [\%] i_{last}$	64.6 10	59.3 15	60.0 15	45.8 10	36.0 15	38.8 15	20.7 10	17.5 15	18.1 15
		$\Phi(0, \alpha_{i_{last}} [\%])$	0.04	0.026	0	0.208	0	0	0.001	0.001	0
		$\tau(0, \alpha_{i_{last}} [\%])$	-93.1	-75.5	-28.3	-67.7	19.4	53.5	103.7	474.9	509
	90	$\alpha_{i_{last}} [\%] i_{last}$	58.7 7	56.8 12	57.2 11	40.7 7	37.4 11	37.8 11	20.5 8	17.4 11	17.4 11
		$\Phi(0, \alpha_{i_{last}} [\%])$	0.015	0	0	0	0	0	0	0.001	0
		$\tau(0, \alpha_{i_{last}} [\%])$	-88.7	-64.3	-0.5	-67.6	-1.9	84.2	103.1	289.8	349.9
	75	$\alpha_{i_{last}} [\%] i_{last}$	54.8 5	53.3 10	53.5 10	37.9 5	35.7 10	36.1 10	19.1 8	17.3 10	17.8 10
		$\Phi(0, \alpha_{i_{last}} [\%])$	0	0	0	0.003	0	0	0.001	0.001	0
		$\tau(0, \alpha_{i_{last}} [\%])$	-86.3	-63.6	-9.6	-70.5	-14.7	39.2	155.8	278.6	337.7

Table S3 Results of numerical experiments after applying the re-aggregation framework to the p-median problem and the benchmark Partizánske.

i	V1			V2			V3		
	α_i [%]	Φ [%]	τ [%]	α_i [%]	Φ [%]	τ [%]	α_i [%]	Φ [%]	τ [%]
1	99.1	15.1	-99	99.1	15.1	-99	99.1	4.12	-98
2	97.1	13.3	-99	96.9	12.8	-98	96.8	2.56	-97
3	90.1	5.57	-98	89.8	4.98	-96	89.3	0.13	-94
4	70.8	1.75	-94	70.7	2.03	-89	68.9	0.11	-83
5	42.8	0.282	-78	45.1	0.25	-65	42.9	0.12	-55
6	27.9	0.084	-50	28.7	0.08	-23	27.2	0	-12
7	22.8	0.018	-15	21.8	0.02	33	21.6	0	42
8	21.0	0.001	31	19.1	0.001	90	19.4	0	100
9	20.7	0.001	67	18.1	0.001	150	18.6	0	159
10	20.7	0.001	103	17.7	0.001	215	18.3	0	219
11	-	-	-	17.6	0.001	278	18.2	0	277
12	-	-	-	17.5	0.001	330	18.1	0	337
13	-	-	-	17.5	0.001	377	18.1	0	394
14	-	-	-	17.5	0.001	423	18.1	0	451
15	-	-	-	17.5	0.001	474	18.1	0	509

Table S4 Performance of the re-aggregation framework in individual iterations after applying it to the p-median problem and benchmark Partizánske for the following parameter values: $\hat{p}=20$, $\hat{\alpha}_1=99$, and $\hat{\varepsilon}=2 km$. Symbol “-“ denotes iterations that were not executed due to termination of the algorithm.

ϵ [unit]	$\hat{\alpha}_1$ [%]	Indicator	$\hat{p}=5$			$\hat{p}=10$			$\hat{p}=20$		
			V1	V2	V3	V1	V2	V3	V1	V2	V3
0	99	$\alpha_{i_{last}} [\%] i_{last}$	99.0 6	85.1 9	96.2 8	99.0 7	85.4 7	99.0 12	98.6 8	59.3 12	99.0 16
		$\Phi_{MAX}(0, \alpha_{i_{last}}) [\%]$	90.385	0	36.538	124.242	6.061	66.667	222.727	0	150
		$\Phi_{GINI}(0, \alpha_{i_{last}}) [\%]$	5.623	-0.916	9.807	26.561	-0.551	13.314	24.754	-1.036	1.027
		$\Phi(0, \alpha_{i_{last}}) [\%]$	-3.171	0.523	-2.132	-1.484	3.651	-0.536	0.345	0.453	-2.929
		$\tau(0, \alpha_{i_{last}}) [\%]$	-100	-100	-100	-100	-100	-99.7	-100	-100	-99.9
	90	$\alpha_{i_{last}} [\%] i_{last}$	90.5 5	90.5 7	88.1 7	87.5 7	76.6 10	90.5 15	81.4 7	73.3 3	90.5 21
		$\Phi_{MAX}(0, \alpha_{i_{last}}) [\%]$	0	0	30.769	6.061	3.03	66.667	9.091	4.545	150
		$\Phi_{GINI}(0, \alpha_{i_{last}}) [\%]$	0.147	0.147	12.619	-0.144	2.879	6.128	1.205	2.808	10.091
		$\Phi(0, \alpha_{i_{last}}) [\%]$	-0.329	-0.329	-4.054	3.239	1.108	-0.492	-0.977	-1.513	-0.457
		$\tau(0, \alpha_{i_{last}}) [\%]$	-100	-100	-99.8	-100	-99.9	-99.8	-100	-100	-99.7
	75	$\alpha_{i_{last}} [\%] i_{last}$	78.6 4	76.1 6	78.6 6	77.8 4	72.5 6	78.6 11	64.3 4	78.6 3	78.6 16
		$\Phi_{MAX}(0, \alpha_{i_{last}}) [\%]$	1.923	0	36.538	6.061	3.03	81.818	0	4.545	150
		$\Phi_{GINI}(0, \alpha_{i_{last}}) [\%]$	-0.542	-0.685	1.81	1.985	0.586	13.284	-1.508	1.413	13.558
		$\Phi(0, \alpha_{i_{last}}) [\%]$	0.338	0.014	0.195	3.3	3.47	10.522	-1.682	-0.914	3.599
		$\tau(0, \alpha_{i_{last}}) [\%]$	-100	-100	-99.6	-100	-99.9	-99.4	-100	-99.9	-99.9
100	99	$\alpha_{i_{last}} [\%] i_{last}$	99.0 6	81.9 10	96.2 9	99.0 10	75.2 11	99.0 9	80.5 11	66.9 5	19.2 30
		$\Phi_{MAX}(0, \alpha_{i_{last}}) [\%]$	90.385	3.846	36.538	124.242	3.03	66.667	59.091	13.636	22.727
		$\Phi_{GINI}(0, \alpha_{i_{last}}) [\%]$	5.623	-0.474	9.807	26.561	0.419	13.314	5.057	-0.251	8.597
		$\Phi(0, \alpha_{i_{last}}) [\%]$	-3.171	0.037	-2.132	-1.484	3.8	-0.536	-3.374	-0.255	2.378
		$\tau(0, \alpha_{i_{last}}) [\%]$	-100	-100	-100	-100	-99.8	-99.2	-100	-100	-99.6
	90	$\alpha_{i_{last}} [\%] i_{last}$	90.5 6	90.5 6	87.6 9	84.9 9	71.8 12	90.5 14	81.0 6	41.4 13	18.1 29
		$\Phi_{MAX}(0, \alpha_{i_{last}}) [\%]$	0	0	30.769	6.061	3.03	66.667	13.636	0	45.455
		$\Phi_{GINI}(0, \alpha_{i_{last}}) [\%]$	0.147	0.147	12.619	1.132	-0.593	6.128	3.735	-0.403	9.83
		$\Phi(0, \alpha_{i_{last}}) [\%]$	-0.329	-0.329	-4.054	4.457	3.836	-0.492	-2.041	0.146	0.723
		$\tau(0, \alpha_{i_{last}}) [\%]$	-100	-100	-99.9	-100	-99.8	-99.1	-99.8	-99.9	-99.7
	75	$\alpha_{i_{last}} [\%] i_{last}$	77.0 4	76.2 5	78.6 6	75.5 5	73.3 4	78.6 4	78.6 6	40.4 12	18.7 24
		$\Phi_{MAX}(0, \alpha_{i_{last}}) [\%]$	0	0	36.538	6.061	6.061	81.818	4.545	0	31.818
		$\Phi_{GINI}(0, \alpha_{i_{last}}) [\%]$	-0.952	-0.588	1.81	0.81	1.995	13.284	1.413	-0.301	11.426
		$\Phi(0, \alpha_{i_{last}}) [\%]$	-0.06	-0.079	0.195	3.375	3.21	10.522	-0.914	0.225	3.15
		$\tau(0, \alpha_{i_{last}}) [\%]$	-100	-100	-99.6	-100	-100	-99.8	-100	-99.9	-99.5
200	99	$\alpha_{i_{last}} [\%] i_{last}$	99.0 8	71.9 12	65.7 21	99.0 10	48.7 10	99.0 15	63.6 12	11.7 12	0.3 14
		$\Phi_{MAX}(0, \alpha_{i_{last}}) [\%]$	90.385	0	34.615	124.242	0	66.667	27.273	0	0
		$\Phi_{GINI}(0, \alpha_{i_{last}}) [\%]$	5.623	0.812	-0.098	26.561	0.395	13.314	3.821	0.286	-2.066
		$\Phi(0, \alpha_{i_{last}}) [\%]$	-3.171	0.106	-4.165	-1.484	2.955	-0.536	-0.007	0.067	0.03
		$\tau(0, \alpha_{i_{last}}) [\%]$	-100	-99.9	-98.5	-99.9	-99.1	-94.4	-99.9	-99.3	-97.7
	90	$\alpha_{i_{last}} [\%] i_{last}$	90.5 6	90.5 7	84.5 8	81.0 6	46.2 11	90.5 13	90.5 7	43.5 8	0.3 12
		$\Phi_{MAX}(0, \alpha_{i_{last}}) [\%]$	0	0	30.769	6.061	0	66.667	13.636	0	0
		$\Phi_{GINI}(0, \alpha_{i_{last}}) [\%]$	0.147	0.147	12.619	0.849	0.051	6.128	6.298	1.781	1.41
		$\Phi(0, \alpha_{i_{last}}) [\%]$	-0.329	-0.329	-4.054	5.478	2.981	-0.492	3.198	0.146	-1.124
		$\tau(0, \alpha_{i_{last}}) [\%]$	-100	-99.9	-99.5	-99.9	-99.3	-98.8	-100	-98.7	-99.7
	75	$\alpha_{i_{last}} [\%] i_{last}$	74.2 5	73.4 6	78.6 7	69.5 6	47.3 7	46.7 12	78.6 9	22.2 7	0.3 11
		$\Phi_{MAX}(0, \alpha_{i_{last}}) [\%]$	0	0	36.538	3.03	0	66.667	4.545	0	0
		$\Phi_{GINI}(0, \alpha_{i_{last}}) [\%]$	-0.952	-0.588	1.81	0.421	-0.384	2.829	1.413	-0.14	-1.061
		$\Phi(0, \alpha_{i_{last}}) [\%]$	-0.06	-0.079	0.195	2.252	2.564	1.208	-0.914	0.026	0.644
		$\tau(0, \alpha_{i_{last}}) [\%]$	-100	-99.9	-97.1	-99.9	-99.8	-98.3	-99.9	-99.8	-99.4

Table S5 Results of numerical experiments after applying the re-aggregation framework to the lexicographic minimax problem and the benchmark d2103.

ϵ [unit]	$\hat{\alpha}_1$ [%]	Indicator	$p=5$			$p=10$			$p=20$		
			V1	V2	V3	V1	V2	V3	V1	V2	V3
0	99	$\alpha_{i_{lost}} [\%] i_{lost}$	97.4 6	86.0 10	91.9 12	96.9 6	74.7 13	95.8 13	94.9 6	58.7 17	64.5 19
		$\Phi_{MAX}(0, \alpha_{i_{lost}}) [\%]$	15.094	1.887	26.415	33.333	0	19.444	76	0	16
		$\Phi_{GINI}(0, \alpha_{i_{lost}}) [\%]$	1.756	2.357	9.834	5.83	0.055	2.463	8.529	-2.905	5.245
		$\Phi(0, \alpha_{i_{lost}}) [\%]$	0.364	-0.044	-0.109	0.076	0.521	-1.758	-3.488	-5.628	-4.049
	$\tau(0, \alpha_{i_{lost}}) [\%]$	-100	-99.9	-100	-100	-100	-100	-100	-100	-100	-100
	90	$\alpha_{i_{lost}} [\%] i_{lost}$	89.6 4	84.9 5	90.1 5	87.7 7	73.1 11	80.8 12	90.1 7	56.6 11	68.6 18
		$\Phi_{MAX}(0, \alpha_{i_{lost}}) [\%]$	7.547	1.887	15.094	5.556	2.778	8.333	12	0	16
		$\Phi_{GINI}(0, \alpha_{i_{lost}}) [\%]$	5.091	0.804	3.31	1.315	0.206	-1.368	1.39	-0.513	4.6
		$\Phi(0, \alpha_{i_{lost}}) [\%]$	1.483	0.263	3.855	-0.112	-0.025	0.908	-6.023	-5.765	-6.245
	$\tau(0, \alpha_{i_{lost}}) [\%]$	-100	-99.9	-99.9	-100	-100	-100	-100	-100	-100	-100
	75	$\alpha_{i_{lost}} [\%] i_{lost}$	74.8 4	72.9 7	75.3 4	75.3 3	62.0 13	73.3 5	75.3 4	54.7 8	75.3 8
		$\Phi_{MAX}(0, \alpha_{i_{lost}}) [\%]$	3.774	1.887	7.547	13.889	2.778	25	8	0	24
$\Phi_{GINI}(0, \alpha_{i_{lost}}) [\%]$		-0.233	-0.039	4.289	-4.836	-2.308	0.789	-0.382	-1.258	5.706	
$\Phi(0, \alpha_{i_{lost}}) [\%]$		0.32	0.912	-0.841	1.534	2.858	-2.135	-5.927	-6.949	-5.528	
$\tau(0, \alpha_{i_{lost}}) [\%]$	-100	-99.9	-99.9	-100	-100	-100	-100	-100	-100	-100	
100	99	$\alpha_{i_{lost}} [\%] i_{lost}$	97.4 9	86.3 10	91.5 15	93.1 9	66.6 15	62.1 19	94.9 7	45.0 13	49.1 23
		$\Phi_{MAX}(0, \alpha_{i_{lost}}) [\%]$	15.094	1.887	26.415	25	0	19.444	76	0	12
		$\Phi_{GINI}(0, \alpha_{i_{lost}}) [\%]$	1.756	1.652	9.465	4.116	-0.618	1.852	8.529	-2.514	5.018
		$\Phi(0, \alpha_{i_{lost}}) [\%]$	0.364	-1.44	-0.094	-0.336	-0.539	-0.814	-3.488	-6.693	-3.426
	$\tau(0, \alpha_{i_{lost}}) [\%]$	-100	-100	-99.9	-100	-100	-99.9	-100	-100	-100	-100
	90	$\alpha_{i_{lost}} [\%] i_{lost}$	87.7 7	85.1 8	90.1 6	87.5 8	71.0 14	83.6 8	85.5 8	43.4 10	72.0 24
		$\Phi_{MAX}(0, \alpha_{i_{lost}}) [\%]$	5.66	1.887	15.094	5.556	0	13.889	8	0	16
		$\Phi_{GINI}(0, \alpha_{i_{lost}}) [\%]$	3.87	0.005	3.31	-0.318	-0.216	-1.98	1.022	0.332	0.196
		$\Phi(0, \alpha_{i_{lost}}) [\%]$	0.45	0.245	3.855	0.953	-0.174	1.638	-5.846	-6.159	-5.323
	$\tau(0, \alpha_{i_{lost}}) [\%]$	-99.9	-100	-99.9	-100	-100	-100	-100	-100	-100	-100
	75	$\alpha_{i_{lost}} [\%] i_{lost}$	75.3 4	70.7 7	75.3 4	73.3 6	60.7 10	72.2 5	75.3 6	50.4 5	75.3 10
		$\Phi_{MAX}(0, \alpha_{i_{lost}}) [\%]$	7.547	0	7.547	8.333	0	25	8	4	24
$\Phi_{GINI}(0, \alpha_{i_{lost}}) [\%]$		0.903	-0.014	4.289	-1.598	-0.709	0.789	-0.382	-0.113	5.706	
$\Phi(0, \alpha_{i_{lost}}) [\%]$		0.85	0	-0.841	0.022	1.257	-2.135	-5.927	-6.439	-5.528	
$\tau(0, \alpha_{i_{lost}}) [\%]$	-99.9	-99.9	-99.9	-100	-100	-100	-100	-100	-100	-100	
200	99	$\alpha_{i_{lost}} [\%] i_{lost}$	96.8 8	79.3 12	98.8 14	96.3 10	57.5 11	95.8 5	93.9 9	53.3 5	33.2 10
		$\Phi_{MAX}(0, \alpha_{i_{lost}}) [\%]$	9.434	1.887	28.302	33.333	0	19.444	76	4	20
		$\Phi_{GINI}(0, \alpha_{i_{lost}}) [\%]$	-0.897	2.171	17.985	5.83	0.018	2.463	8.986	0.323	8.387
		$\Phi(0, \alpha_{i_{lost}}) [\%]$	-1.002	-1.026	5.947	0.076	-0.225	-1.758	-4.19	-5.709	-2.221
	$\tau(0, \alpha_{i_{lost}}) [\%]$	-99.9	-99.9	-99.8	-100	-100	-100	-100	-100	-100	-100
	90	$\alpha_{i_{lost}} [\%] i_{lost}$	84.5 6	73.2 8	75.1 13	85.4 7	59.6 9	90.1 11	90.1 8	55.9 4	28.7 9
		$\Phi_{MAX}(0, \alpha_{i_{lost}}) [\%]$	3.774	0	9.434	8.333	0	16.667	12	4	20
		$\Phi_{GINI}(0, \alpha_{i_{lost}}) [\%]$	2.494	-0.116	2.285	-3.522	-0.216	-1.497	1.39	-1.634	5.881
		$\Phi(0, \alpha_{i_{lost}}) [\%]$	-1.631	0.141	1.98	2.582	-0.174	1.497	-6.023	-5.517	-1.73
	$\tau(0, \alpha_{i_{lost}}) [\%]$	-100	-99.9	-99.7	-100	-100	-99.9	-100	-100	-100	-100
	75	$\alpha_{i_{lost}} [\%] i_{lost}$	75.3 4	65.9 6	75.3 5	75.3 6	45.8 12	55.2 11	75.3 6	75.3 2	30.9 4
		$\Phi_{MAX}(0, \alpha_{i_{lost}}) [\%]$	7.547	0	7.547	13.889	0	11.111	8	8	20
$\Phi_{GINI}(0, \alpha_{i_{lost}}) [\%]$		0.903	-0.116	4.289	-4.836	-0.066	-1.281	-0.382	-0.382	5.385	
$\Phi(0, \alpha_{i_{lost}}) [\%]$		0.85	0.141	-0.841	1.534	-0.34	-0.323	-5.927	-5.927	-5.353	
$\tau(0, \alpha_{i_{lost}}) [\%]$	-100	-99.9	-99.8	-100	-100	-99.7	-100	-100	-100	-100	

Table S6 Results of numerical experiments after applying the re-aggregation framework to the lexicographic minimax problem and the benchmark pcb3038.

ϵ [km]	$\hat{\alpha}_1$ [%]	Indicator	$\hat{p}=5$			$\hat{p}=10$			$\hat{p}=20$		
			V1	V2	V3	V1	V2	V3	V1	V2	V3
0	99	$\alpha_{i_{last}} [\%] i_{last}$	98.0 5	92.3 12	93.8 8	96.5 7	87.9 12	87.4 11	94.6 8	79.7 5	78.3 13
		$\Phi_{MAX}(0, \alpha_{i_{last}}) [\%]$	55.17	2.3	2.3	146.15	0	0	253.13	0	3.13
		$\Phi_{GINI}(0, \alpha_{i_{last}}) [\%]$	-5.81	-19.18	-19.21	3.89	8.75	8.96	18.04	1.33	6.67
		$\Phi(0, \alpha_{i_{last}}) [\%]$	-7.18	7.33	7.33	-11.31	-2.7	-2.66	-2.74	4.4	1.8
		$\tau(0, \alpha_{i_{last}}) [\%]$	-99.36	-93.96	-93.04	-98.52	-80.61	-74.87	-96.29	-44.52	-35.16
	90	$\alpha_{i_{last}} [\%] i_{last}$	89.2 5	86.4 9	88.4 7	88.8 6	86.3 9	86.1 9	87.4 7	80.2 11	78.9 10
		$\Phi_{MAX}(0, \alpha_{i_{last}}) [\%]$	9.2	0	1.15	5.77	1.92	0	21.88	0	0
		$\Phi_{GINI}(0, \alpha_{i_{last}}) [\%]$	-8.36	0.14	3.04	-4.74	1.3	0.61	0.78	-1.67	-0.63
		$\Phi(0, \alpha_{i_{last}}) [\%]$	2.11	-0.26	-0.6	-5.3	-0.31	0.35	-3.97	2.09	2.02
		$\tau(0, \alpha_{i_{last}}) [\%]$	-91.75	-88.89	-83.86	-90.87	-65.3	-60.09	-93.46	-47.7	-22.79
	75	$\alpha_{i_{last}} [\%] i_{last}$	72.5 5	73.3 8	73.2 6	74.4 4	71.6 9	71.1 7	73.5 5	68.2 8	67.5 11
		$\Phi_{MAX}(0, \alpha_{i_{last}}) [\%]$	5.75	0	0	3.85	0	0	15.63	3.13	0
		$\Phi_{GINI}(0, \alpha_{i_{last}}) [\%]$	-14.42	-0.73	-0.55	0.76	0	1.01	-1.22	3	3.48
		$\Phi(0, \alpha_{i_{last}}) [\%]$	7.31	0.25	0.34	-4.35	0.59	0.18	-2.74	3.47	5.41
		$\tau(0, \alpha_{i_{last}}) [\%]$	-68.49	-35.13	-53.52	-72.7	-16.35	-35.74	-59.01	17.84	78.98
1	99	$\alpha_{i_{last}} [\%] i_{last}$	89.4 12	88.0 11	89.0 10	81.8 10	73.3 11	71.3 13	61.2 10	36.3 13	38.9 14
		$\Phi_{MAX}(0, \alpha_{i_{last}}) [\%]$	55.17	2.3	2.3	117.31	0	0	256.25	0	0
		$\Phi_{GINI}(0, \alpha_{i_{last}}) [\%]$	-4.53	-18.8	-19.21	4.17	0	9.4	10.45	1.3	0.89
		$\Phi(0, \alpha_{i_{last}}) [\%]$	-8.26	6.84	7.33	-8.22	0.84	-2.83	-6.99	5.26	4.83
		$\tau(0, \alpha_{i_{last}}) [\%]$	-95.33	-93.68	-89.7	-94.43	-65.04	-41.48	-60.78	359.19	358.66
	90	$\alpha_{i_{last}} [\%] i_{last}$	81.7 8	80.4 8	82.7 8	75.6 7	69.0 10	67.5 9	55.8 7	45.8 10	48.5 11
		$\Phi_{MAX}(0, \alpha_{i_{last}}) [\%]$	11.49	0	0	5.77	0	0	25	0	0
		$\Phi_{GINI}(0, \alpha_{i_{last}}) [\%]$	-7.01	1.31	0.45	-4.97	0.38	1.3	-4	4.11	0.04
		$\Phi(0, \alpha_{i_{last}}) [\%]$	1.35	6.84	7.33	-5.79	0.72	0.06	-0.38	-0.07	4.4
		$\tau(0, \alpha_{i_{last}}) [\%]$	-85.43	-85.47	-83.46	-82	-53.04	-52.26	-43.64	6.36	-4.06
	75	$\alpha_{i_{last}} [\%] i_{last}$	70.8 9	67.9 9	68.4 9	64.5 6	51.1 10	54.5 10	47.5 7	38.4 10	42.0 9
		$\Phi_{MAX}(0, \alpha_{i_{last}}) [\%]$	5.75	0	0	3.85	0	0	15.63	0	0
		$\Phi_{GINI}(0, \alpha_{i_{last}}) [\%]$	-14.93	-0.55	-0.55	1.15	0.27	0.65	-0.41	1.26	0.96
		$\Phi(0, \alpha_{i_{last}}) [\%]$	7.45	0.34	0.34	-4.66	0.3	0.18	-3.68	4.83	5.62
		$\tau(0, \alpha_{i_{last}}) [\%]$	-18.43	2.78	7.36	-39.91	135.39	127.48	113.6	423.85	347.53
2	99	$\alpha_{i_{last}} [\%] i_{last}$	75.2 12	77.2 15	81.2 12	51.7 10	38.7 13	43.7 14	16.6 8	4.7 13	9.0 12
		$\Phi_{MAX}(0, \alpha_{i_{last}}) [\%]$	55.17	2.3	2.3	103.85	0	0	59.38	0	0
		$\Phi_{GINI}(0, \alpha_{i_{last}}) [\%]$	-4.53	-18.83	-19.21	5.69	0.38	0.61	4.59	0.07	0.19
		$\Phi(0, \alpha_{i_{last}}) [\%]$	-8.26	6.84	7.33	-8.39	-0.27	0.35	-1.2	0.14	1.15
		$\tau(0, \alpha_{i_{last}}) [\%]$	-76.98	-72.47	-79.52	-30.61	212	202.52	405.65	1446.64	1008.66
	90	$\alpha_{i_{last}} [\%] i_{last}$	68.9 8	71.3 8	75.3 8	40.4 7	34.0 12	35.7 11	13.8 7	8.2 10	9.8 10
		$\Phi_{MAX}(0, \alpha_{i_{last}}) [\%]$	11.49	2.3	2.3	5.77	0	0	9.38	0	0
		$\Phi_{GINI}(0, \alpha_{i_{last}}) [\%]$	-7.05	-18.83	-19.21	-0.61	0.19	0.61	1.11	0.19	0.22
		$\Phi(0, \alpha_{i_{last}}) [\%]$	1.35	6.84	7.33	-5.3	0.1	0.35	6.27	0.14	1.08
		$\tau(0, \alpha_{i_{last}}) [\%]$	-70.3	-75.9	-79.15	4.17	216.17	199.3	325.09	610.25	503
	75	$\alpha_{i_{last}} [\%] i_{last}$	60.9 9	54.4 9	56.8 9	33.7 7	27.8 10	29.9 9	11.2 7	6.5 9	7.2 9
		$\Phi_{MAX}(0, \alpha_{i_{last}}) [\%]$	5.75	0	0	3.85	0	0	9.38	0	0
		$\Phi_{GINI}(0, \alpha_{i_{last}}) [\%]$	-14.58	-0.45	-0.55	6.5	0.23	0.65	2.93	0	0.11
		$\Phi(0, \alpha_{i_{last}}) [\%]$	7.28	0.06	0.34	-7.36	0.3	0.18	3.75	0	1.37
		$\tau(0, \alpha_{i_{last}}) [\%]$	26.6	43.22	44.47	108.35	468.78	410.87	979.51	1197.7	1051.94

Table S7 Results of numerical experiments after applying the re-aggregation framework to the lexicographic minimax problem and the benchmark Partizánske.

Acknowledgement

This work was supported by the research grants VEGA 1/0463/16 "Economically efficient charging infrastructure deployment for electric vehicles in smart cities and communities", APVV-15-0179 "Reliability of emergency systems on infrastructure with uncertain functionality of critical elements", TRENoP Strategic Research Area and it was facilitated by the FP 7 project ERAdiate [621386]"Enhancing Research and innovation dimensions of the University of Zilina in Intelligent Transport Systems". We thank Dirk Helbing from ETH Zurich for granting the access to the Brutus high-performance cluster and Karl Ernst Ambrosch, Erik Jenelius and Ludmila Jánošíková for valuable comments and suggestions.