Critical behavior in charging of electric vehicles

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Abstract

The increasing penetration of electric vehicles over the coming decades, taken together with the high cost to upgrade local distribution networks and consumer demand for home charging, suggest that managing congestion on low voltage networks will be a crucial component of the electric vehicle revolution and the move away from fossil fuels in transportation. Here, we model the max-flow and proportional fairness protocols for the control of congestion caused by a fleet of vehicles charging on two real-world distribution networks. We show that the system undergoes a continuous phase transition to a congested state as a function of the rate of vehicles plugging to the network to charge. We focus on the order parameter and its fluctuations close to the phase transition, and show that the critical point depends on the choice of congestion protocol. Finally, we analyse the inequality in the charging times as the vehicle arrival rate increases, and show that charging times are considerably more equitable in proportional fairness than in max-flow [1].

The Model

We model the electrical distribution network as a directed rooted tree graph composed of the node set $\mathcal{V}$ and edge set $\mathcal{E}$[2]. Only the root node of the tree injects the power into the network and electric vehicles can be plugged into all other nodes. By the symbol ⋔(j) we denote the subtree rooted in the node $j \in \mathcal{V}$. An edge $e_{ij} \in \mathcal{E}$ connects node i to node j, where i is closer to the root than j, and is characterised by the impedance $Z_{ij} = R_{ij} + iX_{ij}$, where R_{ij} is the edge resistance and X_{ij} the edge reactance. The power loss along edge e_{ij} is given by $S_{ij}(t) = P_{ij}(t) + iQ_{ij}(t)$, where $P_{ij}(t)$ is the real power loss, and $Q_{ij}(t)$ the reactive power loss.

Vehicle l derives a utility $U_l(P_l(t))$ from the allocated charging power $P_l(t)$. By the symbol $V_i(t)$, we denote the voltage level on the node $i \in \mathcal{V}$. We allocate the power to electric vehicles by maximizing the aggregate utility $U(t)$, while making sure that all nodal voltages are within the interval $((1-\alpha)V_{nominal}, (1+\alpha)V_{nominal})$, where α is a parameter and $V_{nominal}$ is the nominal voltage level the network is operated on.

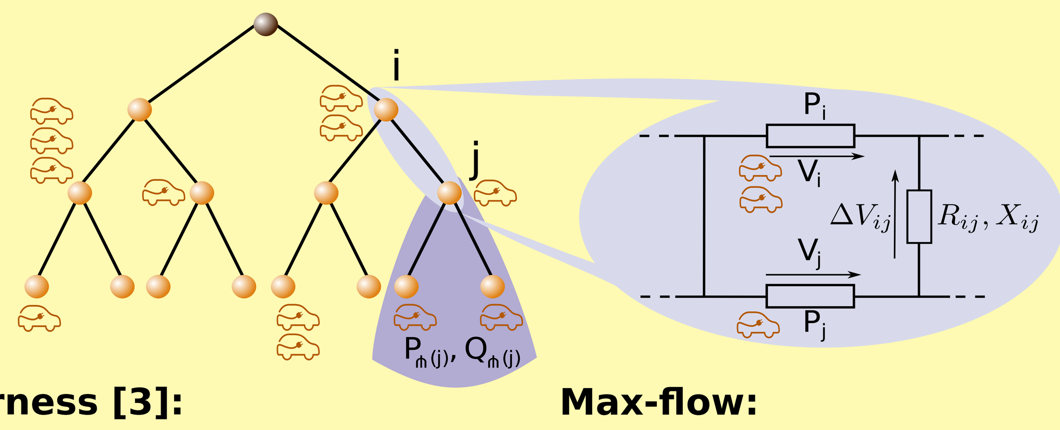
With every edge $e_{ij} \in \mathcal{E}$ is associated one decision variable $W_{ij}(t)$ that it is equal to the product of real voltages on edge nodes, i.e. $W_{ij}(t) = V_i(t)V_j(t)$ and similarly with every node $i \in \mathcal{V}$ is associated variable $W_{ii}(t) = V_i(t)^2$. The generalized inequality (4) means that matrices are positive semidefinite. Constraints (2) ensure that all nodal voltages are within the defined limits. Constraints (3)-(4) encode relations between decision variables $W_{ij}(t)$, power allocations $P_l(t)$ and power losses along edges that arise from Kirchhoff's current and voltage laws:

$$\underset{W(t)}{\text{maximize}} \quad U(t) = \sum_{l=1}^{N(t)} U_l(P_l(t)) \qquad (1)$$

$$\text{subject to} \quad ((1-\alpha)V_{nominal})^2 \le W_{ii}(t) \le ((1+\alpha)V_{nominal})^2 \qquad i \in \mathcal{V} \qquad (2)$$

$$W_{ij}(t) - W_{jj}(t) - P_{\pitchfork(j)}(t)R_{ij} - Q_{\pitchfork(j)}(t)X_{ij} = 0 \qquad e_{ij} \in \mathcal{E} \qquad (3)$$

$$\begin{pmatrix} W_{ii}(t) & W_{ij}(t) \\ W_{ji}(t) & W_{jj}(t) \end{pmatrix} \succeq 0 \qquad e_{ij} \in \mathcal{E} \qquad (4)$$



Proportional fairness [3]:

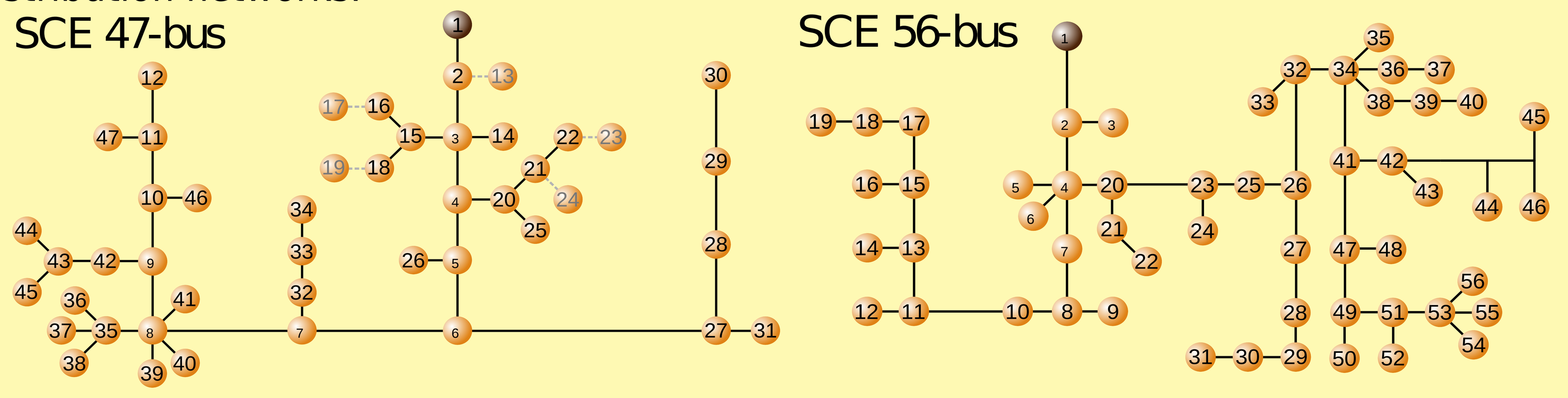
$U(t) = \sum_{l=1}^{N(t)} \log(P_l(t))$

Max-flow:

$U(t) = \sum_{l=1}^{N(t)} P_l(t)$

Numerical Results

To study the behaviour of max-flow and proportional fairness as a function of the number of vehicles arriving at the network to be charged, we implement a discrete simulator that solves the congestion control problem in discrete time steps, starting with no vehicles charging on the network. Vehicles arrive at the network in continuous time (following a Poisson process with rate λ) and with empty batteries, choose a node with uniform probability amongst all nodes (excluding the root), and charge at that node until their battery is full, at which point in time they leave the network. Once a vehicle plugs into a node, the congestion control algorithm will allocate it an instantaneous power, which is a function of the network topology and electrical elements, as well as the location of other vehicles. We simulated vehicles charging on the realistic SCE 47-bus and SCE 56-bus distribution networks.



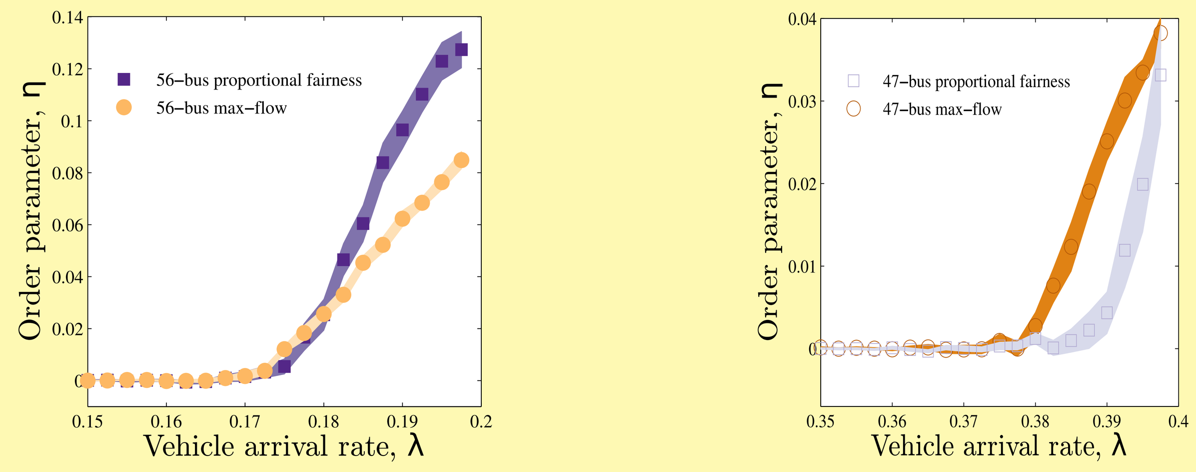
We set $V_{nominal} = B = 1.0$ and $\alpha = 0.1$. In order to characterize the behaviour of the network, we adopt the congestion parameter: $\eta(\lambda) = \lim_{t\to\infty} \frac{1}{\lambda}\frac{\langle \Delta N(t)\rangle}{\Delta t}$,where $\Delta N(t) = N(t+\Delta t) - N(t)$ and $\langle \ldots \rangle$ indicates an average over time window of length Δt. Congestion parameter $\eta(\lambda) = 0$ when all cars leave the network fully charged within a large enough time window, and $\eta(\lambda) > 0$, when some vehicles have to wait for increasingly long times to fully charge, i.e. the network is congested.

The number $N(t)$ of charging vehicles at time t fluctuates widely close to the critical point, and thus it is difficult to determine λ_c .To overcome this limitation, we adopt the susceptibility-like function: $\chi(\lambda) = \lim_{\Delta t\to\infty} \Delta t\ \sigma_\eta(\Delta t)$, where Δt is the length of a time window, and $\sigma_\eta(\Delta t)$ is the standard deviation of the order parameter η .

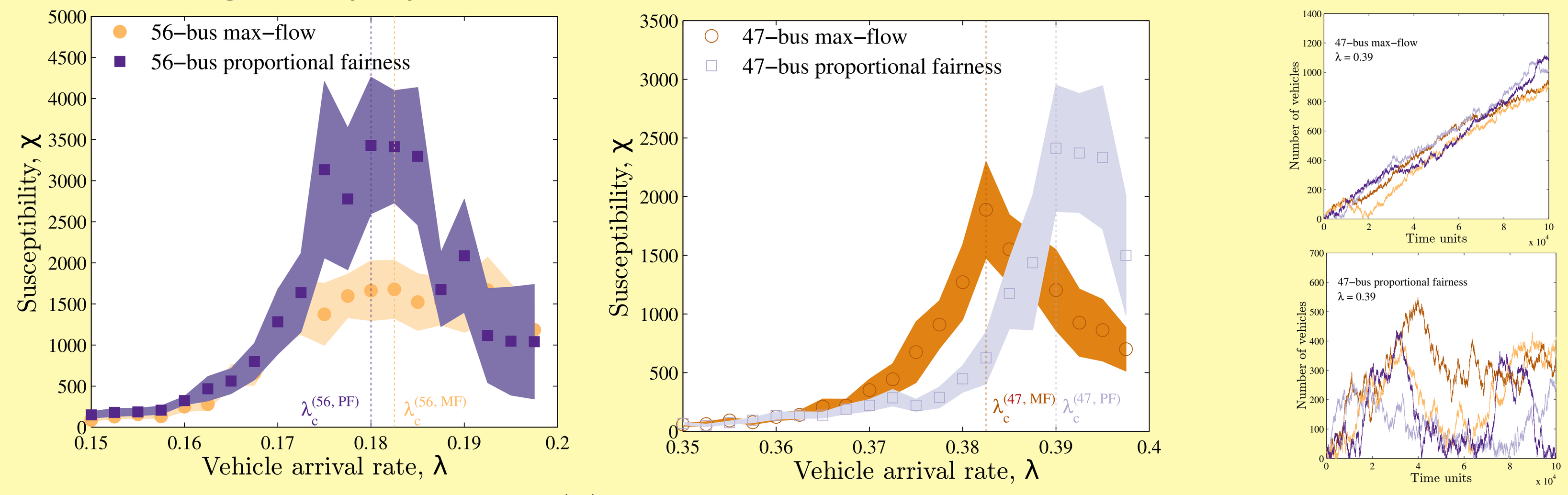
To characterise inequalities in the user experience, we analyse the Gini coefficient of charging time. For a random sample ($x_i, i = 1, 2, \ldots, n$), the empirical Gini coefficient, $\widehat{G}$, may be estimated by a sample mean:

$$\widehat{G} = \frac{\sum_{i=1}^{n}\sum_{j=1}^{n}|x_i - x_j|}{2n^2\mu}.$$

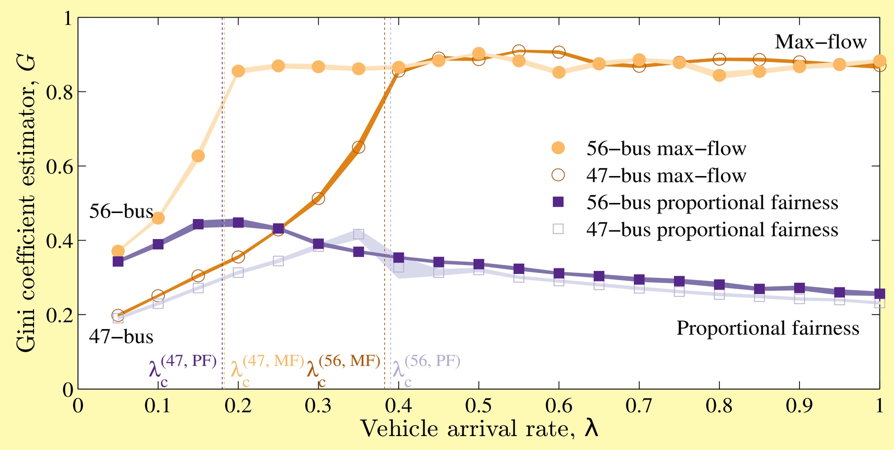
Numerical results



Simulation results suggest that λ_c depends on several factors (the network topology, the complex impedance on the edges, battery capacity, $V_{nominal}$, as well as the position of vehicles on the network). The critical point is numerically indistinguishable for max-flow and proportional fairness in the 56-bus network. In the 47-bus network, however, we find that λ_c is larger for proportional fairness than for max-flow.



Similarly to our analysis of $\eta(\lambda)$, the values of λ_c are indistinguishable in the 56-bus network. In contrast, however, in the 47-bus network the maximum point of $\chi(\lambda)$ is smaller for max-flow than for proportional fairness. This suggests that proportional fairness charges a slightly larger number of vehicles than max-flow, and is thus marginally more efficient, on a neighbourhood of its critical point.



We observe that the Gini coefficient of the charging time is larger in max-flow than in proportional fairness, for each of the networks. Moreover, the Gini coefficient increases faster in max-flow than in proportional fairness in the non-congested regime, showing that, when the system is stable, vehicles will experience a faster increase in the inequality of charging times in max-flow than in proportional fairness, with the increase of the vehicle arrival rate λ_c.

Conclusions

We showed numerically on the 47-bus bus network that the onset of congestion takes place for larger values of λ_c in proportional fairness than in max-flow. We confirmed this results also analytically. This result is surprising, because common λ_c expectation is that efficiency of the system comes at the expense of the increased inequality. However, it should be noted that here we optimise the dynamic system over a certain time period and our optimisation model is not dynamic, hence, it is only a heuristic. Proportional fairness is a promising candidate protocol to manage congestion in the charging of electric vehicles.

Outlooks

Data related challenges: To gain better understanding on how EV drivers use electric vehicles and to refine the demand models of EV charging in residential areas by characterizing more accurately the arrival process, demanded energy and charging profiles desired by users could be derived from data. Suitable data sources are datasets that concen individual mobility of citizens, and operational data from public charging station operators. Available operational data from public charging station operators include (a few thousand) charging stations and (several tens of thousands of) users that are characterized by (millions of) individual charging transactions.

Network model related challenges: The used optimization model could be enhaced to consider both, voltage angles and voltage magnitues and re-formulated as second-order cone programming problem. For simplicity reasons, we did not constraint the power that is assigned to individual cars. Considering it could lead to more realistic behavior, however, the resulting model would pose significant computational challenge.

User model related challenges: The future electrical distribution networks could evolve to support heterogeneous types of loads (network users) using market mechanism. The basic idea is inspired by the resource pricing in communication networks [5]. The network should allow interaction between heterogeneous populations of users by providing mechanisms that could be used to provide active network users with necessary information and the correct incentives to use the network in a fair and efficient way.

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