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Stationary Network Loads May Underestimates Vulnerability to Cascading Failures

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1. Introduction

Our contemporary societies are getting increasingly complex. For instance, a number of infrastructures are critical for their optimal functioning, and their failure can cause devastating consequences. Often cited examples are the electrical power grids; telecommunication networks and the Internet; water, gas and oil distribution pipelines; road, railway and airline transportation networks etc. During the last couple of decades, the public awareness of the importance of their stability and optimal performance has grown significantly. This has in part been prompted by several recent large-scale incidents,in North-America and Europe alike.

State of the power grid shortly before the incident

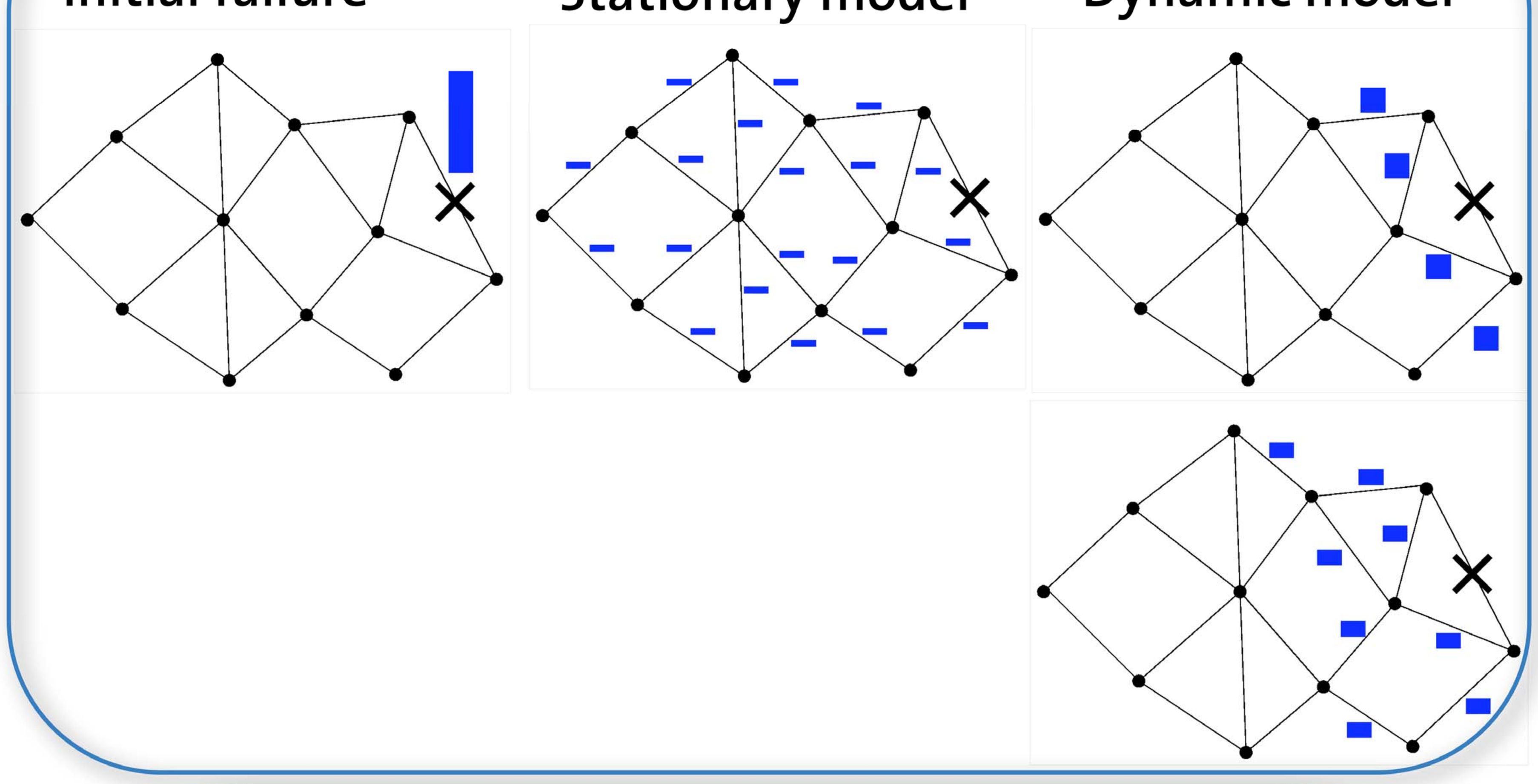
Sequence of events on November 4, 2006

1,3,4,5 – lines switched off for construction work

2 – line switched off for the transfer of a ship by Meyer-Werft

Source : Report on the system incident of November 4, 2006, E.ON Netz GmbH

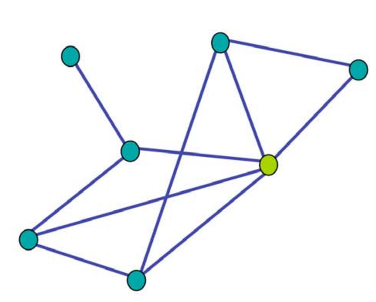
2. Stationary and Dynamic Cascading Models



3. The Modeling Principle (Diffusion Model)

- Random walkers (i.e. particles) "live" on the nodes
- They are moving around on the network!
- In each time step, a walker move *one* step forward towards one of the neighboring vertices chosen by random
- This process is repeated over and over again.......

Note: The number of walkers is constant in time



Network:

$\mathcal{N}$ - set of nodes
$\mathcal{L}$ - set of links
$\boldsymbol{W}$ - adjacency matrix ($W_{ij} \geq 0$ link weight)

Model dynamics: $\boxed{n_i(t+1) = \sum_{j=1}^{\mathcal{N}} T_{ij} n_j(t) + n_i^{\pm}}$ (Master equation)

$n_i(t)$ - number of particles hosted by node *i* at the time *t*

$T_{ij} = W_{ij}/w_j \quad , w_j = \sum_{\ell=1}^{\mathcal{N}} W_{\ell j}$

$n_i^{\pm} > 0$ - node is source, $n_i^{\pm} < 0$ - node is sink

Model normalization:

$\rho_i(t) = n_i(t)/N$ - nodal particle density
$c_i(t) = \rho_i(t)/w_i$ - utilization of outflow current
$j_i^{\pm} = n_i^{\pm}/(N w_i)$ - sinks and sources terms

Dynamic model: → $\boxed{\boldsymbol{c}(t+1) = \boldsymbol{\mathcal{T}} \boldsymbol{c}(t) + \boldsymbol{j}^{\pm}}$ ($\boldsymbol{\tau} = \boldsymbol{T}^T$)

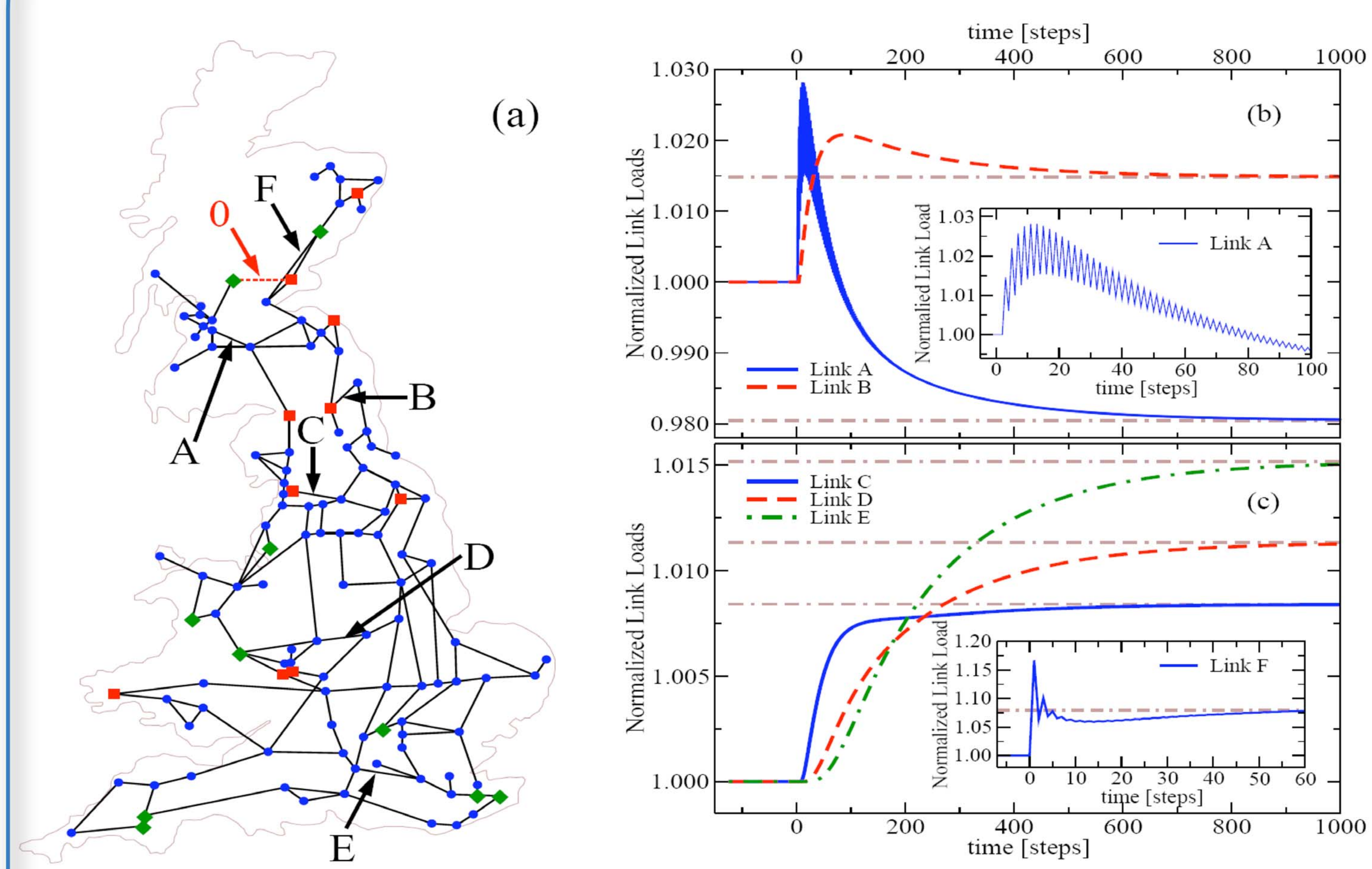
$c_i^{(0)}(\infty) = 1/(N w_i)$ stationary solution for $\boldsymbol{j}^{\pm} = 0$

Stationary model: → $\boxed{\boldsymbol{c}(\infty) = \boldsymbol{c}^{(0)}(\infty) + (\boldsymbol{1} - \boldsymbol{\mathcal{T}})^{+} \boldsymbol{j}^{\pm}}$

$(\boldsymbol{1} - \boldsymbol{\mathcal{T}})^{+}$ generalized inverse of matrix $\boldsymbol{1} - \boldsymbol{\mathcal{T}}$ Link flow: $L_{ij}(t) = C_{ij}(t) + C_{ji}(t)$

$C_{ij}(t) = W_{ij} c_j(t)$ current on link from *j* to *i*

4. Model Dynamics: Example of the UK high voltage power grid (300-400kV)



Green source nodes

Red sink nodes

At t=0, link O is broken!

5. Stationary Model vs. Dynamic Model (the northwestern US power transmission grid)

Link capacities:

$$\boxed{C_{ij} = (1+\alpha)\, L_{ij},}$$

α tolerance parameter

Failure, if:

$$\boxed{C_{ij}(t) > (1+\alpha) L_{ij}}$$

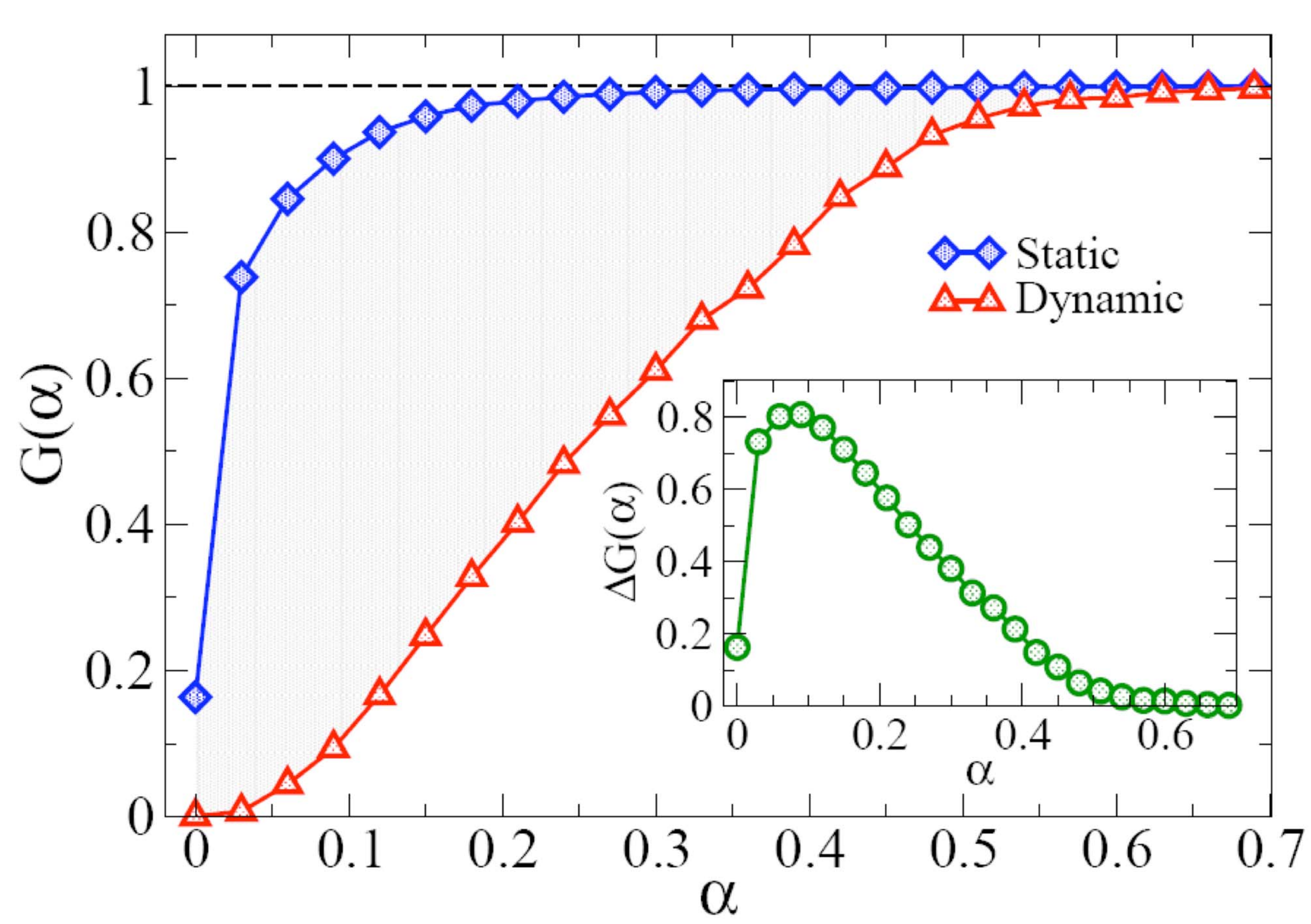
$$G_{\mathcal{L}}(\alpha) = \frac{|\mathcal{L}_R|}{|\mathcal{L}|} \approx G_{\mathcal{N}}(\alpha) = \frac{|\mathcal{N}_R|}{|\mathcal{N}|} = G(\alpha)$$

$|\mathcal{N}|$ - number of nodes (5000)

$|\mathcal{L}|$ - number of links

$|\mathcal{N}_R|$ - number of remaining nodes

$|\mathcal{L}_R|$ - number of remaining links



The stationary model can *overestimates robustness* by more than 80% (in this case)